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Weston M. Stacey

Nuclear Reactor Physics

Third, Revised Edition



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To Penny, Helen, Billy, and Lucia

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Preface

Nuclear reactor physics is the physics of neutron fission chain reacting systems. It encompasses those applications of nuclear physics and radiation transport and interaction with matter that determine the behavior of nuclear reactors. As such, it is both an applied physics discipline and the core discipline of the field of nuclear engineering.

As a distinct applied physics discipline, nuclear reactor physics originated in the middle of the twentieth century in the wartime convergence of international physics efforts in the Manhattan Project. It developed vigorously for roughly the next third of the century in various government, industrial, and university R&D and design efforts worldwide. Nuclear reactor physics is now a relatively mature discipline, in that the basic physical principles governing the behavior of nuclear reactors are well understood, most of the basic nuclear data needed for nuclear reactor analysis have been measured and evaluated, and the computational methodology is highly developed and validated. It is now possible to accurately predict the physics behavior of existing nuclear reactor types under normal operating conditions. Moreover, the basic physical concepts, nuclear data, and computational methodology needed to develop an understanding of new variants of existing reactor types or of new reactor types exist for the most part.

As the core discipline of nuclear engineering, nuclear reactor physics is fundamental to the major international nuclear power undertaking. As of 2000, there are 434 central station nuclear power reactors operating worldwide to produce 350,442 MWe of electrical power. This is a substantial fraction of the world's electrical power (e.g., more than 80% of the electricity produced in France and more than 20% of the electricity produced in the United States). The world's electrical power requirements will continue to increase, particularly as the less developed countries strive to modernize, and nuclear power is the only proven technology for meeting these growing electricity requirements without dramatically increasing the already unacceptable levels of greenhouse gas emission into the atmosphere.

Nuclear reactors have additional uses other than central station electricity production. There are more than 100 naval propulsion reactors in the U.S. fleet (plus others in foreign fleets). Nuclear reactors are also employed for basic neutron physics research, for materials testing, for radiation therapy, for the production of radioisotopes for medical, industrial, and national security applications, and as mobile power sources for remote stations. In the future, nuclear reactors may power deep space missions. Thus, nuclear reactor physics is a discipline important to the present and future well-being of the world.

This book is intended as both a textbook and a comprehensive reference on nuclear reactor physics. The basic physical principles, nuclear data, and computational methodology needed to understand the physics of nuclear reactors are developed and applied to explain the static and dynamic behavior of nuclear reactors in Part 1. This development is at a level that should be accessible to seniors in physics or engineering (i.e., requiring a mathematical knowledge only through ordinary and partial differential equations and Laplace transforms and an undergraduate-level knowledge of atomic and nuclear physics). Mastery of the material presented in Part 1 provides an understanding of the physics of nuclear reactors sufficient for nuclear engineering graduates at the B.S. and M.S. levels, for most practicing nuclear engineers, and for others interested in acquiring a broad working knowledge of nuclear reactor physics.

The material in Part 1 was developed in the process of teaching undergraduate and first-year graduate courses in nuclear reactor physics at Georgia Tech for a number of years. The emphasis in the presentation is on conveying the basic physical concepts and their application to explain nuclear reactor behavior, using the simplest mathematical description that will suffice to illustrate the physics. Numerous examples are included to illustrate the step-by-step procedures for carrying out the calculations discussed in the text. Problems at the end of each chapter have been chosen to provide physical insight and to extend the material discussed in the text, while providing practice in making calculations; they are intended as an integral part of the textbook. Part 1 is suitable for an undergraduate semester-length course in nuclear reactor physics; the material in Part 1 is also suitable for a semester-length first-year graduate course, perhaps with selective augmentation from Part 2.

The purpose of Part 2 is to augment Part 1 to provide a comprehensive, detailed, and advanced development of the principal topics of nuclear reactor physics. There is an emphasis in Part 2 on the theoretical bases for the advanced computational methods of reactor physics. This material provides a comprehensive, though necessarily abridged, reference work on advanced nuclear reactor physics and the theoretical bases for its computational methods. Although the material stops short of descriptions of specific reactor physics codes, it provides the basis for understanding the code manuals. There is more than enough material in Part 2 for a semester-length advanced graduate course in nuclear reactor physics. The treatment is necessarily somewhat more mathematically intense than in Part 1.

Part 2 is intended primarily for those who are or would become specialists in nuclear reactor physics and reactor physics computations. Mastery of this material provides the background for creating the new physics concepts necessary for developing new reactor types and for understanding and extending the computational methods in existing reactor physics codes (i.e., the stock-in-trade for the professional reactor physicist). Moreover, the extensive treatment of neutron transport computational methods also provides an important component of the background necessary for specialists in radiation shielding, for specialists in the applications of neutrons and photons in medicine and industry, and for specialists in neutron, photon, and neutral atom transport in industrial, astrophysical, and thermonuclear plasmas.

Any book of this scope owes much to many people besides the author, and this one is no exception. The elements of the subject of reactor physics were developed by many talented people over the past half-century, and the references can only begin to recognize their contributions. In this regard, I note the special contribution of R.N. Hwang, who helped prepare certain sections on resonance theory. The selection and organization of material has benefited from the example of previous authors of textbooks on reactor physics. The feedback from a generation of students has assisted in shaping the organization and presentation. Several people (C. Nickens, B. Crumbly, S. Bennett-Boyd) supported the evolution of the manuscript through at least three drafts, and several other people at Wiley transformed the manuscript into a book. I am grateful to all of these people, for without them there would be no book.

> WESTON M. STACEY Atlanta, Georgia October 2000

Preface to Second Edition

This second edition differs from the original in two important ways. First, a section on neutron transport methods has been added in Chapter 3 to provide an introduction to that subject in the first section of the book on basic reactor physics that is intended as the text for an advanced undergraduate course. My original intention was to use diffusion theory to introduce reactor physics, without getting into the mathematical complexities of transport theory. I think this works reasonably well from a pedagogical point of view, but it has the disadvantage of sending BS graduates into the workplace without an exposure to transport theory. So, a short section on transport methods in slab geometry was added at the end of the diffusion theory chapter to provide an introduction.

Second, there has been a resurgence in interest and activity in the improvement of reactor designs and in the development of new reactor concepts that are more inherently safe, better utilize the uranium resources, discharge less long-lived waste and are more resistant to the diversion of fuels to other uses. A section has been added in Chapter 7 on the improved Generation-III designs that will be coming online over the next decade or so, and a few sections have been added in Chapters 6 and 7 on the new reactor concepts being developed under the Generation-IV and Advanced Fuel Cycle Initiatives with the objective of closing the nuclear fuel cycle.

The text was amplified for the sake of explication in a few places, some additional homework problems were included, and numerous typos, omissions and other errors that slipped through the final proof-reading of the first edition were corrected. I am grateful to colleagues, students and particularly the translators preparing a Russian edition of the book for calling several such mistakes to my attention.

Otherwise, the structure and context of the book remains unchanged. The first eight chapters on basic reactor physics provide the text for a first course in reactor physics at the advanced undergraduate or graduate level. The second eight chapters on advanced reactor physics provide a text suitable for graduate courses on neutron transport theory and reactor physics.

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I hope that this second edition will serve to introduce to the field the new generation of scientists and engineers who will carry forward the emerging resurgence of nuclear power to meet the growing energy needs of mankind in a safe, economical, environmentally sustainable and proliferationresistant way.

> WESTON M. STACEY Atlanta, Georgia May 2006

Preface to Third Edition

Nuclear reactor physics is that branch of applied nuclear physics that describes the physical behavior of nuclear reactors; as such it is the core discipline of the field of nuclear power engineering. More specifically, nuclear reactor physics describes the physics of the neutron chain fission reaction in a nuclear reactor, which depends on the transport and interaction with matter of fission neutrons and their progeny within a reactor. This physics determines the time-dependent behavior of the neutron distribution in a nuclear reactor and its dependence on the composition and configuration of the materials making up the reactor, which in turn determines the time dependence of the nuclear power level and distribution within the reactor and the change over time in material composition within the reactor due to neutron interactions.

The field of nuclear reactor physics originated in the work of Enrico Fermi, Eugene Wigner, Walter Zinn, and others who designed the Chicago Pile and the Hanford plutonium production reactors in the WWII Manhattan Project in the early 1940s. The early post-war development of the field took place in reactor development programs at what became the Argonne and Oak Ridge National Laboratories, in the naval reactor development Bettis and Knolls Atomic Power Laboratories, and in similar reactor development laboratories abroad (USSR, Canada, France, England, Germany, Japan). Industrial contributions to the field began in the second half of the twentieth century at Westinghouse, General Electric, Combustion Engineering, and Babcock & Wilcox in the United States and subsequently other firms in Europe and Japan, and more recently in China. (The author was privileged to have known and worked with some of the pioneers of the field at the Knolls and Argonne Laboratories.)

This book evolved from lecture notes developed for undergraduate and graduate courses in reactor physics and a graduate course in the related subject of neutron transport theory developed at Georgia Tech in the 1990s. By that time the field had advanced beyond the "bibles" of the field - ANL-5800: *Reactor Physics Constants* and *Naval Reactors Physics Handbook*, and the great early texts on the subject by Weinberg & Wigner, Glasstone & Edlund, Meghreblian & Holmes, LaMarsh, Henry, and Duderstadt & Hamilton were becoming dated, and a lot of new theory had been developed.

My intention in organizing this book was that the first eight chapters would constitute a comprehensive first course in nuclear reactor physics at an advanced undergraduate level. The student would be expected to have some familiarity with the concepts of number densities, cross sections, particle fluxes, radioactivity, and so on going into the course and could expect to come out of the course with a basic knowledge of nuclear reactor physics that would prepare him or her to go on to advanced study of the subject or to take an entry-level job in the nuclear power industry.

Chapters 9-16 are intended for people who are at least acquainted with the material in Chapters 1-8 and want to prepare themselves for advanced nuclear reactor analysis or the development of methods for analyzing new types of nuclear reactors. There is enough material for a graduate course in neutron transport theory (Chapter 9) and more than enough other material (Chapters 10-16) for a graduate course in nuclear reactor physics, and that is the way I have taught it, but of course other arrangements are possible.

Nuclear reactor physics is a math-intensive subject. Understanding of the material in this book would be greatly enhanced by a familiarity with solution of PDEs, by separation of variables and eigenfunction expansion, and by a familiarity with Laplace and Fourier transform methods for the solution of differential equations. This material is usually covered in an advanced undergraduate course in engineering mathematics.

The world certainly needs nuclear power. The climatic threat of continued reliance on fossil fuels and the questionable credibility of deployment of reliable, large-scale baseline solar or wind power plants is authoritatively documented in Burton Richter's *Beyond Smoke and Mirrors*. So, rational maintenance of our standard of living in the developed world and its extension to the remainder of the planet would seem to be dependent on expansion of nuclear power. Improved versions of present reactors and many new variants of reactors are being proposed, which means that many new reactor physics methods must be developed in order to analyze their likely performance. A major purpose of this book is to educate the people who will make these developments and analyze these reactors.

The first and second editions of this book have met with some success, and have been translated into Russian and Chinese. The translators have been familiar with the subject matter, which has resulted in some good questions, and of course they have found some typos and a mistake or two. Similarly, colleagues and readers have identified a few places where a fuller description would be useful. This third edition benefits from their work, which I gratefully acknowledge.

Finally, no book exists without the efforts of the people who produce the physcial product. Martin Preuss stayed after me for a number of years to prepare this third edition, and Stephanie Volk ably edited it at Wiley-VCH. Abhishek Sarkari at Thomson Digital led the copy editors who were essential in pulling together the rather complex final product. I am sincerely grateful to all these people.

WESTON M. STACEY Atlanta, Georgia November, 2017 Part 1

Basic Reactor Physics

Neutron–Nuclear Reactions

The physics of nuclear reactors is determined by the transport of neutrons and their interaction with matter within a reactor. The basic neutron nucleus reactions of importance in nuclear reactors and the nuclear data used in reactor physics calculations are described in this chapter.

1.1 Neutron-Induced Nuclear Fission

Stable Nuclides

Short-range attractive nuclear forces acting among nucleons (neutrons and protons) are stronger than the Coulomb repulsive forces acting among protons at distances on the order of the nuclear radius $(R = 1.25 \times 10^{-13} A^{1/3} \text{ cm})$ in a stable nucleus. These forces are such that the ratio of the atomic mass A (the number of neutrons plus protons) to the atomic number Z (the number of protons) increases with Z; in other words, the stable nuclides become increasingly *neutron-rich* with increasing Z, as illustrated in Fig. 1.1. The various nuclear species are referred to as *nuclides*, and nuclides with the same atomic number are referred to as *isotopes* of the *element* corresponding to Z. We use the notation ${}^{A}X_{Z}$ (e.g., ${}^{235}U_{92}$) to identify nuclides.

Binding Energy

The actual mass of an atomic nucleus is not the sum of the masses (m_p) of the Z protons and the masses (m_n) of A Z neutrons of which it is composed. The stable nuclides have a mass defect:

$$\Delta \quad \begin{bmatrix} Zm_p & A & Zm_n \end{bmatrix} \quad {}^Am_z \tag{1.1}$$

This mass defect is conceptually thought of as having been converted to energy $(E = \Delta c^2)$ at the time that the nucleus was formed, putting the nucleus into a negative energy state. The amount of externally supplied energy that would have to be converted to mass in disassembling a nucleus into its separate nucleons is known as the *binding energy* of the nucleus, $BE = \Delta c^2$. The binding energy per nucleon (BE/A) is shown in Fig. 1.2.



Fig. 1.1 Nuclear stability curve. (With permission from Ref. [1]. Copyright 1996, McGraw-Hill.)



Fig. 1.2 Binding energy per nucleon. (With permission from Ref. [1]. Copyright 1996, McGraw-Hill.)

Any process that results in nuclides being converted to other nuclides with more binding energy per nucleon will result in the conversion of mass into energy. The combination of low *A* nuclides to form higher *A* nuclides with a higher BE/*A* value is the basis for the *fusion* process for the release of nuclear energy. The splitting of very high *A* nuclides to form intermediate-*A* nuclides with a higher BE/*A* value is the basis of the *fission* process for the release of nuclear energy.

Threshold External Energy for Fission

The probability of any nuclide undergoing fission (reconfiguring its *A* nucleons into two nuclides of lower *A*) can become quite large if a sufficient amount of external energy is supplied to excite the nucleus. The minimum, or *threshold*, amount of such *excitation energy* required to cause fission with high probability depends on the nuclear structure and is quite large for nuclides with *Z* < 90. For nuclides with *Z* > 90, the threshold energy is about 4–6 MeV for even-*A* nuclides, and generally is much lower for odd-*A* nuclides. Certain of the heavier nuclides (e.g., ²⁴⁰Pu₉₄ and ²⁵²Cf₉₈) exhibit significant spontaneous fission even in the absence of any externally supplied excitation energy.

Neutron-Induced Fission

When a neutron is absorbed into a heavy nucleus (A, Z) to form a *compound nucleus* (A + 1, Z), the BE/A value is lower for the compound nucleus than for the original nucleus. For some nuclides (e.g., ²³³U₉₂, ²³⁵U₉₂, ²³⁹Pu₉₄, ²⁴¹Pu₉₄), this reduction in BE/A value is sufficient that the compound nucleus will undergo fission, with high probability, even if the neutron has very low energy. Such nuclides are referred to as *fissile*; that is, they can be caused to undergo fission by the absorption of a low-energy neutron. If the neutron had kinetic energy prior to being absorbed into a nucleus, this energy is transformed into additional excitation energy of the compound nucleus. All nuclides with Z > 90 will undergo fission with high probability when a neutron with kinetic energy in excess of about 1 MeV is absorbed. Nuclides such as ²³²Th₉₀, ²³⁸U₉₂, and ²⁴⁰Pu₉₄ will undergo fission with neutrons with energy of about 1 MeV or higher, with high probability.

Neutron Fission Cross Sections

The probability of a nuclear reaction, in this case fission, taking place can be expressed in terms of a quantity σ that expresses the probable reaction rate per unit area normal to the neutron motion for *n* neutrons traveling with speed v, a distance *dx* in a material with *N* nuclides per unit volume:

$$\sigma \quad \frac{\text{reaction rate}}{nvNdx} \tag{1.2}$$

The units of σ are area that gives rise to the concept of σ as a cross-sectional area presented to the neutron by the nucleus, for a particular reaction process, and to the designation of σ as a *cross section*. Cross sections are usually on the order of 10²⁴ cm², and this unit is referred to as a *barn*, for historical reasons.

6 1 Neutron–Nuclear Reactions

The fission cross section, σ_{f} is a measure of the probability that a neutron and a nucleus interact to form a compound nucleus that then undergoes fission. The probability that a compound nucleus will be formed is greatly enhanced if the relative energy of the neutron and the original nucleus, plus the reduction in the nuclear binding energy, corresponds to the difference in energy of the ground state and an excited state of the compound nucleus, so that the energetics are just right for formation of a compound nucleus in an excited state. The first excited states of the compound nuclei resulting from neutron absorption by the odd-*A* fissile nuclides are generally lower lying (nearer to the ground state) than the first excited states of the compound nuclei resulting from neutron absorption by the heavy even-*A* nuclides, which accounts for the odd-*A* nuclides having much larger absorption and fission cross sections for low-energy neutrons than do the even-*A* nuclides.

Fission cross sections for some of the principal fissile nuclides of interest for nuclear reactors are shown in Figs. 1.3–1.5. The resonance structure corresponds to the formation of excited states of the compound nuclei, the lowest lying of which are at less than 1 eV. The nature of the resonance cross section can be shown to give rise to a $1/E^{1/2}$ or 1/v dependence of the cross section at off-resonance neutron energies below and above the resonance range, as is evident in these figures. The fission cross sections are largest in the thermal energy region $E < \sim 1 \text{ eV}$. The thermal fission cross section for $^{239}\text{Pu}_{94}$ is larger than that of $^{235}\text{U}_{92}$ or $^{233}\text{U}_{92}$.

Fission cross sections for $^{238}U_{92}$ and $^{240}Pu_{94}$ are shown in Figs. 1.6 and 1.7. Except for resonances, the fission cross section is insignificant below about 1 MeV, above which it is about 1 barn. The fission cross sections for these and other even-*A* heavy mass nuclides are compared in Fig. 1.8, without the resonance structure.



Fig. 1.3 Fission cross sections for ²³³U₉₂. (From www.nndc.bnl.gov/.)



Fig. 1.4 Fission cross sections for ²³⁵U₉₂. (From www.nndc.bnl.gov/.)



Fig. 1.5 Fission cross sections for ²³⁹Pu₉₄. (From www.nndc.bnl.gov/.)

Products of the Fission Reaction

A wide range of nuclides are formed by the fission of heavy mass nuclides, but the distribution of these fission fragments is sharply peaked in the mass ranges 90 < A < 100 and 135 < A < 145, as shown in Fig. 1.9. With reference to the